MATH L.0.8

QENA STUDENT CLUB

Concepts

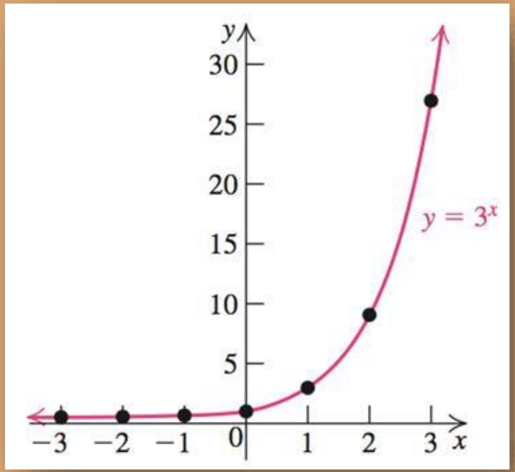
- EXPONENTIAL FUNCTION
- LOGARITHMIC FUNCTION
- GROWTH
- DECAY

EXPONENTIAL FUNCTIONS

- The exponential function with base a is defined by
- $f(x)=a^x$
- where $a>0$, $a\neq 1$, and x is a real number.
- If the base were a negative number, the value of the function would be a complex number for some values of x.
- is defined such that $a\neq 1$because $f(x)=1^x=1$ is a constant function

Properties of exponential functions of the form $fx=a\hat{}x$,

- 1.The function is a one-to-one function as the domain of the function is $(-\infty,\infty)$and the range of the function is $(0,\infty)$.
- 2.The graph of f is a smooth, continuous curve with a y-intercept of $(0,1)$, and the graph passes through $(1,a)$.
- 3.The graph of $f(x)=a\hat{}x$ has no x-intercepts, so it never crosses the x-axis. No value of x will cause $f(x)=a\hat{}x$ to equal 0.
- 4.The x-axis is a horizontal asymptote for every exponential function of the form $f(x)=x\hat{}a$

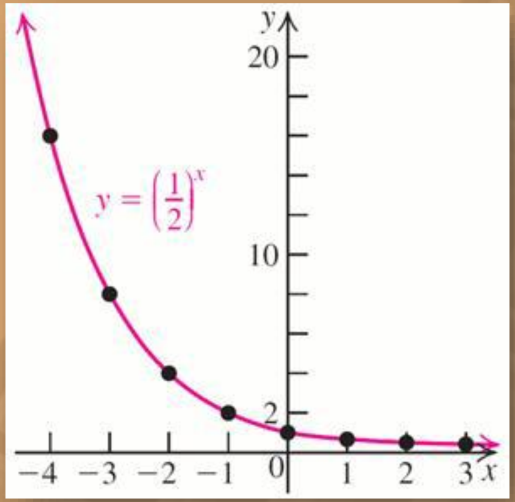


For $a>1$, **Exponential Growth**

f is an increasing function, so the graph rises to the right.

As $x\to\infty$, $y\to\infty$

As $x\to-\infty$, $y\to0$



For $0<a<1$, -**Exponential Decay**

f is a decreasing function, so the graph falls to the right.

As $x\to-\infty$, $y\to\infty$

As $x\to\infty$, $y\to0$

Natural Exponential Function

-The irrational number $e$is useful in many applications that involve growth or decay.
-The letter e represents the number that $(1+1/n)^n$ approaches as n increases without bound.
-The value of $e$accurate to eight decimal places is 2.71828183.

Natural exponential Function

-For all real numbers x, the function defined by
$f(x)=e^x$

LOGARITHMIC FUNCTIONS

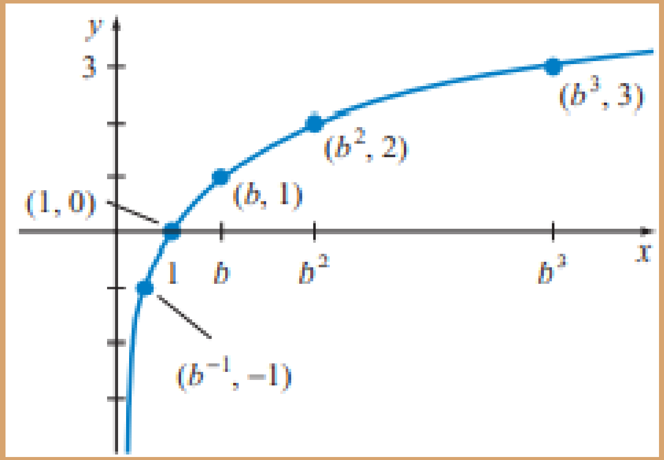
- If $x>0$and b is a positive constant ($b\neq 1$), then
- $y=\log b$ X if and only if $b^y=x$
- The notation is read “the logarithm (or log) base b of x.” The function defined by $f(x)=\log b$ X is a **logarithmic function** with base b. This function is the inverse of the **exponential function** $g(x)=b^x$
- **Composition of Logarithmic and Exponential Functions**
- Let $g(x)=b^x$ and $f(x)=\log b$ X ($x>0,b>0,b\neq 1$). Then
- $g(f(x))=b$ ^$\log b$ X$=x$ and $f(g)x))=$ $\log b$ B^$x=x$
- Notes:
- The **exponential form** of $y=\log b$ X is $by=x$.
- The **logarithmic form** of $by=x$ is $y=\log b$ X.

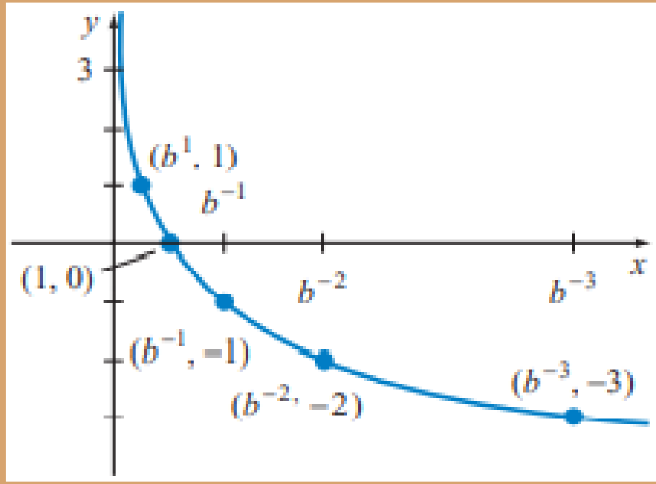
PROPERTIES OF F(X)=LOG BX

1.The domain of the function is $(0, \infty)$and the range of the function is $(-\infty, \infty)$.

- 2.The graph of $f$has an x-intercept of $(1,0)$and passes through $(b,1)$.
- 3. **If $b>0$, f** is an increasing function and its graph is asymptotic to the **negative y-axis**.
- $x \to \infty$, $f(x) \to \infty$
- $x \to 0$, $f(x) \to -\infty$

- 4. **If $0<b<1$, f**is an decreasing function and its graph is asymptotic to the **positive y-axis**.
- $x \to \infty$, $f(x) \to -\infty$
- $x \to 0$, $f(x) \to \infty$





PERCENT INCREASE AND DECREASE

You can model growth or decay by a constant percent increase or percent decrease with the formula:

Initial Amount

$$A(t) = a\,(1 \pm r)t$$

Number of Time Periods

Rate of Increase

Final Amount

- 1+r is growth factor
- 1-r is decay factor

Growth & Decay

Growth : is when data rises over a period of time, creating an upwards trending curve on a graph. In mathematics

Decay : Is process in which a quantity decreases over time, with the rate of decrease becoming proportionally smaller as the quantity gets smaller.

Exponential Growth And Decay

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Exponential Growth	Exponential Decay
$f(x) = a(1 + r)^t$	$f(x) = a(1 - r)^t$

r - rate of growth

t - time steps

Example 1: What is the amount received from the investment fund after 2 years, if $.100,000 were invested at the compounding rate of 5% per every quarter?

Solution:

- The invested principal is a = $100,000, the rate of compounding growth is r = 5% = 0.05 per quarter.
- The time period is 2 years, and there are 4 quarters in a year, and we have t = 8.
- Applying the concepts of exponential growth and decay we have the following expressions for exponential growth.
- $f(x) = a(1 + r)^t$
- $f(x) = 100,000(1 + 0.05)^8$
- $f(x) = 1,00,000(1.05)^8 = 100,000 \times 1.47745544 = 147745.44$
- Therefore an amount of $1,47, 746 is received after a period of 2 years.

Example 2: The radioactive material of thorium decays at the rate of 8% per minute. What part of 10 grams of thorium would be remaining after 5 minutes?

Solution:

- The given initial quantity of thorium is a = 10grams, the rate of decay per minute is r = 8% = 8/100 = 0.08, and the time steps t = 5.
- Here we can apply the concepts of exponential growth and decay, and the exponential decay formula for the decay of thorium is as follows.
- $f(x) = a(1 - r)^t$
- $^f(x) = 10(1 - 0.08)^5 = 10(0.92)^5 = 6.5908$
- Therefore a quantity of 6.6 grams of thorium remains after 5 minutes.

THANKS

Made by:
Qena Student Club